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Inflation without quantum gravity

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Abstract. It is sometimes argued that observation of tensor modes from inflation would provide the first evidence for quantum gravity. However, in the usual inflationary formalism, also the scalar modes involve quantised metric perturbations. We consider the issue in a semiclassical setup in which only matter is quantised, and spacetime is classical. We assume that the state collapses on a spacelike hypersurface, and find that the spectrum of scalar perturbations depends on the hypersurface. For reasonable choices, we can recover the usual inflationary predictions for scalar perturbations in minimally coupled single-field models. In models where non-minimal coupling to gravity is important and the field value is sub-Planckian, we do not get a nearly scale-invariant spectrum of scalar perturbations. As gravitational waves are only produced at second order, the tensor-to-scalar ratio is negligible. We conclude that detection of inflationary gravitational waves would indeed be needed to have observational evidence of quantisation of gravity.

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1 Introduction

Inflation and the quantisation of gravity. The recent observation by the BICEP2 experiment [1] of B-mode polarisation of the cosmic microwave background in excess of the signal due to lensing [2], at first credited to gravitational waves generated during inflation, turned out to be due to Galactic foregrounds [3, 4]. (There are also other possible cosmological sources of B-modes, such as magnetic fields [5], topological defects [6] and self-ordering scalar fields [7], though these could not have explained the signal.)

The claimed detection of inflationary tensor modes was hailed as the first observational confirmation of quantum gravity, because inflationary tensor perturbations are generated from quantum fluctuations of the metric¹. Since inflationary gravitational waves have not been observed, does this mean that there is no observational evidence for quantum gravity? In the usual treatment of inflation, this is not the case, because scalar perturbations of the metric are also quantised. More precisely, the relevant scalar quantity is the Sasaki-Mukhanov variable [9], which is the linear combination of matter and metric perturbations that satisfies canonical commutation relations. (Neither the gauge-invariant scalar metric perturbation nor the gauge-invariant scalar field perturbation satisfy canonical commutation relations on their own; see e.g. [10].) The resulting predictions for the spectrum of scalar modes are in excellent agreement with observations. However, if it is possible to obtain the same results by quantising only the matter variables, then gravitational waves would really be needed to establish quantisation of the metric. The issue is complicated by the fact that we do not know which approximation to quantum gravity is valid during inflation.

Semiclassical quantum gravity. Inflation is the only area of physics where it has been possible to observationally probe the interface between general relativity and quantum theory [11, 12]. Indeed, it seems that an indeterministic theory is required to explain the origin of inhomogeneity and anisotropy in the universe. Deterministically it is only possible to refer anisotropy and inhomogeneity to an earlier inhomogeneous and anisotropic state, absent

¹Note that observation of primordial gravitational waves with an amplitude of 10^{-5} would not, in itself, prove quantisation of gravity, unlike argued in [8]. First order scalar perturbations generate tensor perturbations at second order, so if the amplitude of scalar perturbations were 10^{-3} to 10^{-2} , gravitational waves with an amplitude of 10^{-5} would be generated by this classical process. Arguments based on dimensional analysis and the amplitude of primordial gravitational waves alone do not provide evidence for quantum gravity.

constraints that only allow a unique initial state or dynamical laws with preferred locations and directions. The indeterminism of quantum mechanics makes it possible to break the symmetry of the initial state by collapse, in which only one member of the statistically homogeneous and isotropic initial distribution of possibilities is realised. This connects the generation of inflationary perturbations to state collapse in the primordial universe.

There is no complete theory of quantum gravity, only different approximate or speculative formulations. Quantisation of metric perturbations in the Einstein-Hilbert action (with added higher order curvature counterterms) around a classical background leads to a non-renormalisable theory. However, it is possible for perturbatively non-renormalisable theories to be non-perturbatively well-defined [13]. In the case of gravity, one possibility is asymptotic safety, which means that the renormalisation group flow has a fixed point corresponding to a finite-dimensional hypersurface in the space of coupling constants [14, 15]. Even non-renormalisable theories may be treated at low energies using effective field theory [14, 16] and perturbative quantum gravity is no exception [17]. Quantum corrections can be organised in a series of higher order curvature terms, possibly along with important infrared effects not captured by such an expansion [18, 19].

Quantising perturbations of the metric around a classical background, as done in the usual treatment of inflation, is often called the semiclassical approximation. However, the term semiclassical is also used to refer to a distinct approach to quantum field theory on curved spacetime, in which quantum matter evolves in a classical spacetime [20]. This formalism has been developed to great sophistication in terms of algebraic quantum field theory [21]. In this approach, quantum fields are coupled to the classical metric via the expectation value of the energy-momentum tensor being proportional to the Einstein tensor. We use this theory to study whether it is possible to reproduce the successes of the usual inflationary formalism without quantising the metric. If it turns out that the usual inflationary predictions for the scalar modes can be reproduced without quantising metric perturbations, then observation of inflationary gravitational waves would (in the inflationary context) be necessary in order to conclude that gravity is quantised. (We are here neglecting the issue of loop corrections due to gravitons, which are below present observational sensitivity.)

This is not expected to be a valid approximation close to the Planck scale or when the variance is large and typical realisations are far from the mean (see also [22]). Another issue is that the expectation value of the energy-momentum tensor, and thus the metric, changes discontinuously when the state collapses, which can lead to problems. For example, if state collapse can be precipitated locally, causality can be violated, as this makes it possible to send signals with infinite speed. (If the metric is also in a superposition state until collapse, there is no such problem.) We will only consider a single collapse event, and do not specify the collapse mechanism. Also, in inflation the amplitude of typical perturbations is small, so different realisations are not far from each other. Whether or not the domain of validity of the semiclassical approximation extends to inflation can only be settled by a more complete theory of quantum gravity or by direct comparison to observation. Inflation without quantised metric perturbations has been previously considered in [23].

We take the distribution of perturbations in the initial state to be homogeneous and isotropic. The expectation value of the energy-momentum tensor is then spatially homogeneous and isotropic, and the spacetime is taken to be the homogeneous and isotropic Friedmann–Robertson–Walker (FRW) universe. Once the state collapses into a specific realisation of the quantum mechanical distribution, the expectation value of the energy-momentum tensor is no longer homogeneous and isotropic, and will source metric perturba-

tions. We treat the system classically after the collapse, so the quantum mechanical distribution of realisations becomes a classical distribution in space, as usual for inflation. Unlike in the usual treatment, the resulting power spectrum of the curvature perturbation depends on the details of the collapse. As usual in quantum mechanics, we assume that the state collapses on a spacelike hypersurface, and we investigate different choices of hypersurface. (See [23–25] for scenarios where the collapse happens at different times for different wavemodes.)

In section 2 we give the action and the equations of motion in the semiclassical formalism. In section 3 we consider matching across the hypersurface of collapse and calculate the spectrum for different choices. We show that for minimally coupled single field inflation models, we can recover the usual predictions for scalar perturbations, up to slow-roll suppressed corrections. If non-minimal coupling to gravity is important, we do not get a nearly scale-invariant spectrum (at least for sub-Planckian field values). We summarise our results in section 4.

2 Semiclassical inflation

2.1 Action and equations of motion

Classical case. We consider scalar field matter that may be non-minimally coupled to gravity,

$$S = \int d^4x \sqrt{-g} \left(\frac{M^2 + \xi \varphi^2}{2} R - \frac{1}{2} g^{\alpha\beta} \partial_\alpha \varphi \partial_\beta \varphi - V(\varphi) \right), \quad (2.1)$$

where M and ξ are constants and R is the Ricci scalar. We use the metric signature $(-+++)$, the conventions of [26] for the curvature functions, and the sign convention where $\xi = -\frac{1}{6}$ is the conformally coupled case. If both matter and spacetime were classical, we would obtain the equations of motion by varying the action (2.1) with respect to the metric and the scalar field.

If the metric is taken to be the only independent gravity variable, variation of (2.1) with respect to the metric gives

$$G_{\alpha\beta} = \frac{1}{M^2} T_{\alpha\beta}, \quad (2.2)$$

where $G_{\alpha\beta}$ is the Einstein tensor and $T_{\alpha\beta}$ is the energy-momentum tensor, and the energy-momentum tensor is

$$T_{\alpha\beta} = S_{\alpha\beta} - \xi (G_{\alpha\beta} - \nabla_\alpha \nabla_\beta + g_{\alpha\beta} \square) \varphi^2, \quad (2.3)$$

where $S_{\alpha\beta} \equiv \partial_\alpha \varphi \partial_\beta \varphi - g_{\alpha\beta} [\frac{1}{2} g^{\gamma\delta} \partial_\gamma \varphi \partial_\delta \varphi + V(\varphi)]$. In the Palatini formulation, where the metric and the connection are independent variables, the energy-momentum tensor is different from (2.3), unless the non-minimal coupling ξ vanishes. We come back to this difference when we consider Higgs inflation in section 3.2. The energy-momentum tensor (2.3) is quadratic in the field, and thus well suited to quantisation. For cosmological analysis, it is more convenient to replace $G_{\alpha\beta}$ using (2.2) to obtain [27]

$$T_{\alpha\beta} = \frac{M^2}{M^2 + \xi \varphi^2} [S_{\alpha\beta} + \xi (\nabla_\alpha \nabla_\beta - g_{\alpha\beta} \square) \varphi^2], \quad (2.4)$$

The effective Planck mass corresponds to $\sqrt{M^2 + \xi\varphi^2}$, so for $|\xi|\varphi^2 \ll M^2$, it is close to M . We adopt units such that $M = 1$.

Variation of (2.1) with respect to the scalar field gives

$$\square\varphi + \xi R\varphi - \frac{dV}{d\varphi} = 0 . \quad (2.5)$$

Semiclassical case. We consider quantum matter in a classical spacetime. The relation between the two is given by the semiclassical Einstein equation

$$G_{\alpha\beta} = \langle T_{\alpha\beta} \rangle , \quad (2.6)$$

where $\langle T_{\alpha\beta} \rangle$ is the expectation value of the energy-momentum tensor. We neglect higher order curvature terms, which arise when renormalising a quantum field in curved spacetime. For small curvature, their effect is small, apart from the feature that they change the order of the differential equations and typically destabilise the solutions [28]. However, instabilities related to higher derivatives are presumably beyond the range of validity of our semiclassical approach [29]².

We assume that the expectation value of the energy-momentum tensor before the collapse is homogeneous and isotropic and the spacetime is FRW. We do not consider how inflation has started and how the spacetime has become homogeneous and isotropic [31–33]. This inverts the usual relation between the symmetry of the background space and the symmetry of the quantum mechanical distribution. Usually the distribution inherits homogeneity and isotropy from the background space, whereas in our case spacetime is FRW because of the symmetry of the distribution³. We assume that inflation has lasted sufficiently long that spatial curvature can be neglected, so the metric is

$$ds^2 = a(\eta)^2 [-d\eta^2 + \delta_{ij}dx^i dx^j] . \quad (2.7)$$

We denote $\mathcal{H} \equiv a'/a$, where prime denotes derivative with respect to conformal time η .

We write the field operator in terms of a homogeneous expectation value and perturbation as $\hat{\varphi} = \bar{\varphi} + \delta\hat{\varphi}$, with $\langle \hat{\varphi} \rangle = \bar{\varphi}$. We neglect quantum corrections to the equations of motion, which are small during inflation at least in chaotic inflation models [34], which we consider in section 3.2. Expanding the action (2.1) around $\bar{\varphi}$ to quadratic order in perturbations gives

$$S = \int d^4x \sqrt{-g} \left(\frac{1+\xi\bar{\varphi}^2}{2} R - \frac{1}{2} g^{\alpha\beta} \partial_\alpha \bar{\varphi} \partial_\beta \bar{\varphi} - V(\bar{\varphi}) \right) - \frac{1}{2} \int d^4x \sqrt{-g} \delta\hat{\varphi} \left(-\square - \xi R + \frac{d^2 V(\bar{\varphi})}{d\bar{\varphi}^2} \right) \delta\hat{\varphi} . \quad (2.8)$$

The field perturbation is written in terms of annihilation and creation operators $\hat{a}_{\mathbf{k}}$ and $\hat{a}_{\mathbf{k}}^\dagger$ (respectively) as usual,

$$\delta\hat{\varphi}(\eta, \mathbf{x}) = \sum_{\mathbf{k}} \left(\hat{a}_{\mathbf{k}} u_{\mathbf{k}}(\eta, \mathbf{x}) + \hat{a}_{\mathbf{k}}^\dagger u_{\mathbf{k}}^*(\eta, \mathbf{x}) \right) . \quad (2.9)$$

(Strictly speaking, (2.9) is not a well-defined operator and has to be smeared over a space-time patch to obtain a well-defined quantity [21].) We assume that the spacetime allows a

²See [30] for an example of a class of theories with infinitely high derivatives that have a well-defined initial value problem.

³In the first inflationary model, the initial state was actually assumed to be homogeneous and isotropic [11].

decomposition in terms of \mathbf{k} -modes. As we restrict our attention to FRW spacetime, this is not a problem, and we have (with some abuse of notation) $u_{\mathbf{k}}(\eta, \mathbf{x}) = u_{\mathbf{k}}(\eta)e^{i\mathbf{k}\cdot\mathbf{x}}$.

From (2.8) we get the equations of motion for $\bar{\varphi}$ and $u_{\mathbf{k}}$ by varying with respect to $\bar{\varphi}$ and $\delta\hat{\varphi}$,

$$\bar{\varphi}'' + 2\mathcal{H}\bar{\varphi}' - 6\xi(\mathcal{H}' + \mathcal{H}^2)\bar{\varphi} + a^2 \frac{dV(\bar{\varphi})}{d\bar{\varphi}} = 0 \quad (2.10)$$

$$u_{\mathbf{k}}'' + 2\mathcal{H}u_{\mathbf{k}}' - 6\xi(\mathcal{H}' + \mathcal{H}^2)u_{\mathbf{k}} + a^2 \frac{d^2V(\bar{\varphi})}{d\bar{\varphi}^2}u_{\mathbf{k}} = 0, \quad (2.11)$$

where in (2.11) we only consider modes with super-Hubble wavelengths. The normalisation of $u_{\mathbf{k}}$ is fixed by the fact that $\delta\hat{\varphi}$ satisfies canonical commutation relations⁴. A crucial feature of the semi-classical mode equation (2.11) is that during inflation for a minimally coupled theory it coincides with the fully quantised result up to slow-roll corrections⁵.

An important question is in which state the expectation value is taken. We follow the usual assumption that the system is in the Bunch-Davies vacuum, which is a Hadamard state. As the Hamiltonian does not commute with the field operator, this vacuum is not an eigenstate of the field operator, so the field does not have a well-defined value. This is the origin of inflationary quantum fluctuations.

Instead of expanding the action around a non-zero vacuum expectation value, we could assume that the expectation value of the field in the Bunch-Davies vacuum $|0\rangle$ is zero, $\hat{\varphi} = \delta\hat{\varphi}$. In this case, we could obtain a non-vanishing expectation value by taking the system to be in a coherent state instead. Specifically, we can consider the state $\hat{U}|0\rangle \equiv e^{N\hat{a}_0 + N^*\hat{a}_0^\dagger}|0\rangle$, where \hat{a}_0 is the annihilation operator corresponding to the zero mode and $N(\eta)$ is a function chosen so that $\langle\hat{\varphi}\rangle = \langle\delta\hat{\varphi}\rangle = 2\text{Re}(Nu_0)$ satisfies (2.5), so we have $\langle\hat{\varphi}\rangle = \bar{\varphi}$, as before. (In other words, the time-dependence of N is adjusted by hand to correct the evolution of the zero mode to follow the full non-linear equation of motion (2.10) instead of its linearisation (2.11).) We can go back to the situation with a non-zero vacuum expectation value by transforming the field instead of transforming the state (i.e. switching from the Schrödinger representation to the Heisenberg representation), which gives $\delta\hat{\varphi} \rightarrow \hat{U}^\dagger\delta\hat{\varphi}\hat{U} = 2\text{Re}(Nu_0) + \delta\hat{\varphi} = \bar{\varphi} + \delta\hat{\varphi}$. However, the situation with the coherent state is not physically fully equivalent to the case where we start from the action (2.8), because the time evolution of the coherent state function N is not determined by the classical equations of motion.

Either way, we have

$$\begin{aligned} \langle\hat{\varphi}^n\rangle &= \sum_{m=0}^{[n/2]} \frac{n!}{(2m)!(n-2m)!} \bar{\varphi}^{n-2m} \langle 0|\delta\hat{\varphi}^{2m}|0\rangle \\ &= \sum_{m=0}^{[n/2]} \frac{n!}{2m!(n-2m)!} \bar{\varphi}^{n-2m} \left(\int_0^\infty \frac{dk}{k} \mathcal{P}_{\delta\hat{\varphi}}(k) \right)^m, \end{aligned} \quad (2.12)$$

⁴As mentioned in section 1, in the usual treatment of inflation, the gauge-invariant field perturbation does not satisfy canonical commutation relations, nor does the gauge-invariant scalar metric perturbation. As the two are related by a constraint, it would be impossible for both of them to satisfy canonical commutation relations; see e.g. [10].

⁵This feature that the semiclassical and fully quantum treatments agree up to small corrections is not generic: For example in the usual treatment of the hydrogen atom one must invoke Gauss's law as an operator identity and not as a relation between two expectation values in order to obtain correct results.

where $[n/2]$ is the integer part of $n/2$. On the second line we have assumed that the statistics of the quantum fluctuations are Gaussian, homogeneous and isotropic and written $\langle 0|\delta\hat{\varphi}^2|0\rangle = \int_0^\infty \frac{dk}{k} \mathcal{P}_{\delta\hat{\varphi}}(k)$, where $\mathcal{P}_{\delta\hat{\varphi}}$ is the power spectrum of the vacuum fluctuations of $\delta\hat{\varphi}$. If the typical amplitude of the power spectrum is small and it decays at both small and large k , so that $\int_0^\infty \frac{dk}{k} \mathcal{P}_{\delta\hat{\varphi}}(k)$ is small, we have $\langle 0|\delta\hat{\varphi}^2|0\rangle \ll \bar{\varphi}^2$. If the time derivatives of the mode functions are not much larger than the time derivatives of $\bar{\varphi}$, the expectation value of the energy-momentum tensor in (2.6) reduces to the classical form (2.3) in terms of $\bar{\varphi}$ (Strictly speaking, the situation is more complex, and the argument requires renormalising divergent quantities, and depends on the form of the inflaton potential [34]⁶.)

2.2 From homogeneity and isotropy to perturbations

Metric and equations of motion. Before the collapse, the field is in a superposition state of different field values, and its probability distribution is homogeneous and isotropic. When the state collapses, the field takes an almost definite value at each point in space, and homogeneity and isotropy are broken. (If the field value were exactly definite, its time derivative would be completely indefinite due to the uncertainty principle. For classical-looking states, the probability distribution of both the field and its time derivative is highly peaked, but not exactly definite.) The expectation value of the energy-momentum tensor jumps discontinuously to having perturbations, which source perturbations in the metric. Scalar field matter does not source vector and tensor perturbations at first order, so we only have scalar perturbations, and the metric can be written as

$$ds^2 = a(\eta)^2 \left[-(1 + 2A)d\eta^2 + 2B_{,i}d\eta dx^i + (\delta_{ij} - 2\delta_{ij}\psi + 2E_{,ij})dx^i dx^j \right], \quad (2.14)$$

and we denote $C \equiv B - E'$. After the collapse, we treat the expectation value of the quantum field as a classical quantity as per usual, and we split it into background plus perturbation, $\langle \hat{\varphi} \rangle \equiv \varphi = \bar{\varphi}(\eta) + \delta\varphi(\eta, \mathbf{x})$. (Note the slight difference of notation from the pre-collapse era: in both cases, $\bar{\varphi}$ denotes a homogeneous background field, but after the collapse, the expectation value of the field has perturbations around the mean.) The background evolution is given by (2.10) together with the Einstein equation (2.2) for the background,

$$3\mathcal{H}^2 = \frac{1}{1 + \xi\bar{\varphi}^2} \left(\frac{1}{2}\bar{\varphi}'^2 + a^2V - 6\xi\mathcal{H}\bar{\varphi}\bar{\varphi}' \right). \quad (2.15)$$

⁶From (2.8) we get the quantum correction to the energy-momentum tensor as (using cosmic time instead of conformal time; dot denotes derivative with respect to cosmic time)

$$\begin{aligned} \langle \hat{T}_{00}^Q \rangle &= \sum_{\mathbf{k}} \left\{ \frac{1}{2} \left[|\dot{v}_{\mathbf{k}}|^2 + \left(\frac{\mathbf{k}^2}{a^2} + \frac{d^2V(\bar{\varphi})}{d\bar{\varphi}^2} \right) |v_{\mathbf{k}}|^2 \right] - \xi \left[G_{00} + 3\frac{\dot{a}}{a}\partial_0 \right] |v_{\mathbf{k}}|^2 \right\} \\ &= \sum_{\mathbf{q}} \left\{ b_1(\mathbf{q}) + b_2(\mathbf{q})G_{00} + b_3(\mathbf{q})\frac{d^2V(\bar{\varphi})}{d\bar{\varphi}^2} + \mathcal{O}(\dot{v}_{\mathbf{q}}) \right\}, \end{aligned} \quad (2.13)$$

where $\mathbf{q} \equiv \mathbf{k}/a$. The coefficients $b(\mathbf{q})$ depend on $v_{\mathbf{q}}$, and we have used the normalized mode functions $v_{\mathbf{k}} \equiv u_{\mathbf{k}}a^{-3/2}$ in de Sitter space [20]. When $\dot{u}_{\mathbf{q}}$ is small, the $b(\mathbf{q})$'s give (divergent) constants that can be absorbed in the bare action, provided that $V(\bar{\varphi})$ is a polynomial in $\bar{\varphi}$. The first two terms in the second line of (2.13) are absorbed in the cosmological constant and Newton's constant, respectively, and the rest contribute to the matter action.

For the perturbations, the Einstein equation (2.2) and the field equation of motion (2.5) written in terms of Fourier modes reduce to (we only consider super-Hubble modes)

$$\begin{aligned} \delta\varphi''_{\mathbf{k}} + 2\mathcal{H}\delta\varphi'_{\mathbf{k}} - 6\xi(\mathcal{H}' + \mathcal{H}^2)\delta\varphi_{\mathbf{k}} + a^2\frac{d^2V}{d\bar{\varphi}^2}\delta\varphi_{\mathbf{k}} = -2a^2\frac{dV}{d\bar{\varphi}}A_{\mathbf{k}} + \bar{\varphi}'(A'_{\mathbf{k}} + 3\psi'_{\mathbf{k}} - k^2C_{\mathbf{k}}) \\ - \xi\bar{\varphi}(6\psi''_{\mathbf{k}} + 18\mathcal{H}\psi'_{\mathbf{k}} + 6\mathcal{H}A'_{\mathbf{k}} + k^2[4\psi_{\mathbf{k}} - 2A_{\mathbf{k}} - 2C'_{\mathbf{k}} - 6\mathcal{H}C_{\mathbf{k}}]) \end{aligned} \quad (2.16)$$

$$\psi_{\mathbf{k}} = A_{\mathbf{k}} + C'_{\mathbf{k}} + 2\mathcal{H}C_{\mathbf{k}} + 2\frac{\xi\bar{\varphi}}{1 + \xi\bar{\varphi}^2}(\delta\varphi_{\mathbf{k}} + \bar{\varphi}'C_{\mathbf{k}}) \quad (2.17)$$

$$\psi'_{\mathbf{k}} + \mathcal{H}A_{\mathbf{k}} = \frac{1}{1 + \xi\bar{\varphi}^2} \left(\frac{1}{2}\bar{\varphi}'\delta\varphi_{\mathbf{k}} + \xi[\bar{\varphi}'\delta\varphi_{\mathbf{k}} - \mathcal{H}\bar{\varphi}\delta\varphi_{\mathbf{k}} + \bar{\varphi}\delta\varphi' - \bar{\varphi}\bar{\varphi}'A_{\mathbf{k}}] \right) \quad (2.18)$$

$$\begin{aligned} \mathcal{H}C_{\mathbf{k}} = \frac{1 + \xi\bar{\varphi}^2}{1 + \xi\bar{\varphi}^2 + \xi\mathcal{H}^{-1}\bar{\varphi}\bar{\varphi}'}\psi_{\mathbf{k}} \\ + \frac{1}{2} \frac{1 + \xi\bar{\varphi}^2 + 6\xi^2\bar{\varphi}^2}{(1 + \xi\bar{\varphi}^2)(1 + \xi\bar{\varphi}^2 + \xi\mathcal{H}^{-1}\bar{\varphi}\bar{\varphi}')} \left(\frac{\bar{\varphi}'}{k} \right)^2 \left([\eta_H - \epsilon_H]\mathcal{H}\frac{\delta\varphi_{\mathbf{k}}}{\bar{\varphi}'} + \frac{\delta\varphi'_{\mathbf{k}}}{\bar{\varphi}'} - A_{\mathbf{k}} \right), \end{aligned} \quad (2.19)$$

where we have defined $\epsilon_H \equiv -(\mathcal{H}' - \mathcal{H}^2)/\mathcal{H}^2$, $\eta_H \equiv 1 - \bar{\varphi}''/(\mathcal{H}\bar{\varphi}') + \epsilon_H$. We also denote $\eta_{H2} \equiv 2 - \bar{\varphi}'''/(\mathcal{H}^2\bar{\varphi}') + 2\epsilon_H - 3\eta_H$.⁷ We have not assumed that these quantities are small. In slow-roll (and for minimal coupling) they reduce to the usual slow-roll parameters ϵ , η and $\xi_2 - 4\epsilon^2 - \eta^2 + \epsilon\eta$, respectively. The comoving curvature perturbation on large scales is (for proof of conservation in the case $\xi = 0$, see e.g. [35], section 2.2)

$$\mathcal{R} = -\psi - \mathcal{H} \frac{[(1 + 2\xi)\bar{\varphi}' - 2\xi\mathcal{H}\bar{\varphi}]\delta\varphi + 2\xi\bar{\varphi}\delta\varphi' - 2\xi\bar{\varphi}\bar{\varphi}'A}{(1 + 2\xi)\bar{\varphi}'^2 - 2\xi\mathcal{H}\bar{\varphi}\bar{\varphi}'(1 - \epsilon_H + \eta_H)}. \quad (2.20)$$

In the case $\xi = 0$, this reduces to the usual result $\mathcal{R} = -\psi - \mathcal{H}\delta\varphi/\bar{\varphi}'$.

3 Matching across the collapse

3.1 Hypersurface of collapse

Squeezing, decoherence and collapse. During inflation quantum modes are stretched to super-Hubble wavelengths and become squeezed as the ratio of the constant and decaying solutions of the mode equation grows exponentially [36]. The modes decohere and the quantum mechanical distribution takes the shape of a classical stochastic distribution [37]. However, it is not understood how an approximately classical, definite-seeming universe emerges, i.e. how the state collapses, though presumably this happens after squeezing and decoherence. Some suggested collapse theories have been applied to inflation, but the matter remains unsettled [23–25]. Note that the issue of decoherence (suppression of quantum mechanical interference terms) is different from that of collapse (definite outcome).

In the usual formulation of inflation where both metric and matter perturbations are quantised, the quantum mechanical distribution of the Sasaki-Mukhanov variable (or the comoving curvature perturbation \mathcal{R} , treated as a quantum variable) is simply equated with its classical distribution, without considering the collapse process. We instead equate the pre-collapse quantum mechanical power spectrum of the field with the post-collapse power spectrum of the classical field, $\mathcal{P}_{\hat{\varphi}} = \mathcal{P}_{\varphi}$ or, equivalently, $\mathcal{P}_{\delta\hat{\varphi}} = \mathcal{P}_{\delta\varphi}$, on some spacelike

⁷In terms of cosmic time t , $\epsilon_H = -\partial_t H/H^2$, $\eta_H - \epsilon_H = -\partial_t^2 \bar{\varphi}/(H\partial_t \bar{\varphi})$ and $\eta_{H2} = -\partial_t^3 \bar{\varphi}/(H^2\partial_t \bar{\varphi})$, where $H \equiv \mathcal{H}/a$.

hypersurface of collapse. The field is discontinuous at collapse, but its power spectrum is continuous, and the power spectrum of its time derivative can also be taken to be continuous, so we can match $\mathcal{P}_{\delta\dot{\varphi}} = \mathcal{P}_{\delta\varphi'}$. Discontinuity of the metric perturbations implies, via (2.16), that the power spectra of second and higher order time derivatives of the field are discontinuous across the collapse.

Unlike in [23–25], collapse conditions are not defined for individual modes in momentum space, but instead all wavelengths collapse simultaneously. Squeezing depends on the wavelength of the mode, so sub-Hubble modes will not have become squeezed and decohered at the time of collapse, and their distribution could be very different from the classical stochastic case. Wavelengths of such modes are extremely small compared to present-day cosmological scales, as inflation typically lasts for a few dozen e-folds after the observable modes become super-Hubble. It is not clear whether their non-classical distribution would have any observable consequences. Possible signals from very small scales include primordial black holes [38] and gravitational waves.

In contrast to the usual inflationary treatment, the power spectrum of the comoving curvature perturbation \mathcal{R} depends on which hypersurface the collapse happens. In the pre-collapse era, space is exactly homogeneous and isotropic, and the slice can be defined in terms of Killing vectors, but after the collapse there is no exact symmetry, and the division into an FRW background slice plus perturbations is simply a matter of convenience [39]. On the collapse hypersurface, the unique exactly homogeneous and isotropic pre-collapse spatial slice is matched onto one of the infinite number of possible background post-collapse spatial slices. Equivalently, we have to specify on which hypersurface the classical variable $\delta\varphi$ is defined. This ambiguity is related to the fact that $\delta\varphi$ is not gauge-invariant. However, the choice of the hypersurface of collapse is not a gauge choice, and different hypersurfaces lead to different power spectra for \mathcal{R} . Viewed physically, we have to specify which physical quantity is constant on the hypersurface of collapse.

Choice of hypersurface. In the absence of a description of the collapse process, we simply consider different spatial hypersurfaces of collapse and see how the power spectrum of the comoving curvature perturbation depends on the choice. In principle, it is possible to get any desired power spectrum by choosing the appropriate ψ on the hypersurface of collapse. Nevertheless, some choices of hypersurface seem more natural than others. In this subsection, we put $\xi = 0$, because we will see in section 3.2 that the kind of hypersurfaces we consider do not lead to a nearly scale-invariant spectrum if the non-minimal coupling is important, except possibly in the case $|\xi| \ll 1$, $|\bar{\varphi}| \gg 1$. We specify a hypersurface by setting conditions on the metric or on some physical quantities. From those conditions we solve for ψ in terms of $\delta\varphi$ and $\delta\varphi'$ using (2.16)–(2.19), and use (2.20) to obtain \mathcal{R} in terms of $\delta\varphi$ and $\delta\varphi'$. There are two unknowns, A and C . If the hypersurface condition fixes C (and does not contain inverse spatial derivatives of $\delta\varphi$ and $\delta\varphi'$), we obtain, in the super-Hubble limit,

$$\mathcal{R}_{\mathbf{k}} = -(1 + \eta_H - \epsilon_H)\mathcal{H}\frac{\delta\varphi_{\mathbf{k}}}{\bar{\varphi}'} - \frac{\delta\varphi'_{\mathbf{k}}}{\bar{\varphi}'} - C'_{\mathbf{k}} - 2\mathcal{H}C_{\mathbf{k}} , \quad (3.1)$$

where all quantities are evaluated at the time of collapse, not at Hubble crossing. If the hypersurface condition fixes A instead, we obtain

$$\mathcal{R}_{\mathbf{k}} = \frac{1}{\epsilon_H}(f'_{\mathbf{k}} + 2\mathcal{H}f_{\mathbf{k}}) , \quad (3.2)$$

Constant	Curvature perturbation \mathcal{R}_k
$C = 0$	$-(1 - \chi_1)\mathcal{H}\frac{\delta\varphi_k}{\varphi'} - \frac{\delta\varphi'_k}{\varphi'}$
θ	$-3\epsilon_H\frac{\mathcal{H}^2}{k^2}\left[(3 + \chi_1)\mathcal{H}\frac{\delta\varphi_k}{\varphi'} + \frac{\delta\varphi'_k}{\varphi'}\right]$
$\sigma = 0$	$-2(1 - \chi_1)\mathcal{H}\frac{\delta\varphi_k}{\varphi'} - 2\frac{\delta\varphi'_k}{\varphi'}$
$\dot{\varphi}$	$-\frac{\mathcal{H}^2}{k^2}\left[(3\chi_1 + \chi_2)\mathcal{H}\frac{\delta\varphi_k}{\varphi'} + \chi_1\frac{\delta\varphi'_k}{\varphi'}\right]$
ρ	$3\frac{\mathcal{H}^2}{k^2}\left[(3 + \chi_1)\mathcal{H}\frac{\delta\varphi_k}{\varphi'} + \frac{\delta\varphi'_k}{\varphi'}\right]$
p	$-\frac{\mathcal{H}^2}{k^2}\left[(9 + 9\chi_1 + 2\chi_2)\mathcal{H}\frac{\delta\varphi_k}{\varphi'} + (3 + 2\chi_1)\frac{\delta\varphi'_k}{\varphi'}\right]$
R	$-3\frac{\mathcal{H}^2}{k^2}\left[(6 + 5\chi_1 + \chi_2)\mathcal{H}\frac{\delta\varphi_k}{\varphi'} + (2 + \chi_1)\frac{\delta\varphi'_k}{\varphi'}\right]$
$^{(3)}R = 0$	$-2\mathcal{H}\frac{\delta\varphi_k}{\varphi'}$

Table 1. Curvature perturbation for different choices of what is kept constant (in the case of C , σ and $^{(3)}R$, kept zero) on the hypersurface of collapse. All quantities are evaluated at the time of collapse. Here we taken $\xi = 0$ and denoted $\chi_1 \equiv \epsilon_H - \eta_H$ and $\chi_2 \equiv \epsilon_H\chi_1 - \eta_{H2}$.

where $f_k \equiv \epsilon_H\mathcal{H}k^{-2}\left([\eta_H - \epsilon_H]\mathcal{H}\frac{\delta\varphi_k}{\varphi'} + \frac{\delta\varphi'_k}{\varphi'} - A_k\right)$. Again, all quantities are evaluated at the time of collapse. Note that in this case \mathcal{R} will depend on $\delta\varphi''$, unless A contains exactly one factor of $\frac{\delta\varphi'}{\varphi'}$. As the power spectrum of $\delta\varphi''$ is discontinuous at collapse, such a hypersurface will not yield a well-defined power spectrum.

We first consider the simple possibility of matching $\mathcal{P}_{\delta\hat{\varphi}} = \mathcal{P}_{\delta\varphi_{\text{GI}}}$, where $\delta\varphi_{\text{GI}} \equiv \delta\varphi + \dot{\varphi}'C$ is the gauge invariant field perturbation. This corresponds to setting $C = 0$, so if the collapse happens during slow-roll, and perturbations evolve slowly, $|\delta\varphi'_k| \ll \mathcal{H}|\delta\varphi_k|$, (3.1) shows that the power spectrum of \mathcal{R} in terms of $\delta\varphi$ is the same as in the usual inflationary case in the spatially flat gauge, $\psi = 0$. In the spatially flat gauge the field equation of motion also reduces to (2.11) at zeroth order in slow-roll. We therefore obtain the same power spectrum as in the usual case, up to slow-roll suppressed corrections.

Covariant quantities. Let us now consider collapse hypersurfaces defined in terms of covariant physical quantities. Consider a frame-independent physical scalar quantity S . The unit vector orthogonal to the hypersurface of constant S , assumed to be spacelike, is

$$n^\alpha = \frac{1}{\sqrt{-g^{\gamma\delta}\partial_\gamma S\partial_\delta S}}g^{\alpha\beta}\partial_\beta S. \quad (3.3)$$

It has been argued that in single-field inflation the preferred time variable is the field [40]. This would correspond to taking the collapse hypersurface to be the one of constant φ . However, the condition $\delta\varphi = 0$ does not give a well-defined spectrum for \mathcal{R} on the hypersurface of collapse (and if it did, the result would be zero). We can instead keep

constant some other quantity defined in terms of n^α for the choice $S = \varphi$. The gradient of n^α can be decomposed in terms of the expansion rate θ , shear $\sigma_{\alpha\beta}$ and acceleration \dot{n}^α , where dot refers to derivative along n^α [41]. From $\sigma_{\alpha\beta}$ and \dot{n}^α , we can form the scalar quantities $\sigma \equiv \sqrt{\frac{1}{2}\sigma^{\alpha\beta}\sigma_{\alpha\beta}}$ and $\dot{n} \equiv \sqrt{\dot{n}^\alpha\dot{n}_\alpha}$. Some simple choices of quantities to keep constant on the hypersurface are $\dot{\varphi}$, constant expansion rate θ , shear σ , spatial curvature scalar ${}^{(3)}R$, energy density ρ or pressure p . In the case of σ and ${}^{(3)}R$, they are actually zero on the hypersurface. Taking the spacetime Ricci scalar R to be constant defines a hypersurface in a frame-independent way (i.e. without having to specify n^α). Obviously, other choices are possible, but those that involve $\delta\varphi''$ or derivatives of metric perturbations, which are not defined at the moment of collapse, will not yield a power spectrum for \mathcal{R} . Examples of such ill-defined choices include the hypersurface of constant volume acceleration $\dot{\theta} + \frac{1}{3}\theta^2$ or constant (in fact, zero) acceleration \dot{n} (taking $S = \varphi$ in both cases).

In table 1, we give results for the above choices of collapse hypersurface. We have two kinds of modifications to the spectrum compared to the usual inflationary case. The first possibility is that the spectrum is multiplied by the factor $(k/\mathcal{H})^{-4}$, evaluated at collapse. Without inflation producing a very blue spectrum for $\delta\varphi$, with a spectral index close to $n = 5$, the resulting spectrum for \mathcal{R} is not close to scale-invariant, in strong disagreement with observations. Also, for observable modes we have $k \ll \mathcal{H}$ at collapse, and the amplitude of $\mathcal{P}_{\mathcal{R}}$ would be exponentially larger than the amplitude of $\mathcal{P}_{\delta\varphi}$. In addition to $\mathcal{P}_{\delta\dot{\varphi}} = \mathcal{P}_{\delta\varphi_{\text{GI}}}$, this leaves the hypersurfaces of constant (in fact, vanishing) σ and ${}^{(3)}R$ as possibly viable choices.

The other modification is a uniform change in the amplitude. The magnitude depends on the hypersurface of collapse and on when the collapse occurs. Presumably the state does not collapse before modes in the observable range have become squeezed and decohered, and it may be that the collapse does not happen until the decay of the inflaton field. Conceivably, it could even happen later, but we only follow the system up to the beginning of inflaton oscillations, and do not consider decay. The change in the amplitude is determined by the values of $\epsilon_H - \eta_H$, $\mathcal{H}\delta\varphi/\dot{\varphi}'$ and $\delta\varphi'/\dot{\varphi}'$ at collapse, so it depends on the inflationary model. We first look at two simple minimally coupled inflationary models, and then consider the non-minimally coupled case.

3.2 Inflation models

Minimal coupling. As examples, we consider the minimally coupled ($\xi = 0$) inflation models defined by the potentials $V = \frac{1}{2}m^2\varphi^2$ and $V = \frac{1}{4}\lambda\varphi^4$. The background equations (2.10) and (2.15) are the same as in the usual inflationary formalism, but the mode equation (2.11) is different from the usual equation (2.16) due to the absence of metric perturbations. However, as discussed above, in the usual inflationary case the contribution of metric perturbations to (2.16) vanishes in the spatially flat gauge ($\psi = 0$) to zeroth order in slow-roll. Therefore we get essentially the same results for $\mathcal{P}_{\delta\varphi}$ as in the usual case, if the state collapses during slow-roll. If we match $\mathcal{P}_{\delta\dot{\varphi}} = \mathcal{P}_{\delta\varphi_{\text{GI}}}$ across the collapse, we also get the usual result for $\mathcal{P}_{\mathcal{R}}$, up to slow-roll suppressed corrections. For the hypersurface of constant σ or ${}^{(3)}R$, the amplitude of $\mathcal{P}_{\mathcal{R}}$ is increased by a factor of two compared to the usual case.

If the state collapses when the field oscillates after inflation, the absolute value of the amplitude can change up or down by an arbitrary amount, for any of these three hypersurfaces. (Obviously, the range of validity of our perturbative calculation does not extend to non-perturbatively large amplitudes, so the most we can say is that the amplitude can

become non-perturbatively large.) In figure 1 we show \mathcal{R} as a function of N_c , the number of e-folds until the end of inflation at the moment of collapse (so $N_c = 0$ corresponds to $\epsilon_H = 1$), normalised to the usual spatially flat gauge result $-\mathcal{H}\delta\varphi/\bar{\varphi}'$ evaluated at 60 e-foldings until the end of inflation. At the top we show the result for $\mathcal{P}_{\delta\bar{\varphi}} = \mathcal{P}_{\delta\varphi_{\text{GI}}}$ (for the hypersurface of constant σ , the result is multiplied by 2, as seen in table 1). If the collapse happens deep into inflation, the change is small, but if the state collapses when the inflaton field is oscillating, the amplitude can change up or down by any amount, because $\bar{\varphi}'$ passes through zero. For the potential $V = \frac{1}{2}m^2\varphi^2$, shown on the left side, \mathcal{R} first diverges at $N_c = -0.7$, and then undergoes divergent oscillations with a negative amplitude. For $V = \frac{1}{4}\lambda\varphi^4$, the behaviour is similar, with \mathcal{R} first diverging at $N_c = -1.5$, and then undergoing divergent oscillations with a positive amplitude. At the bottom we show the result for collapse on the hypersurface of constant $^{(3)}R$. The behaviour is similar, with the amplitude undergoing divergent oscillations.

If the amplitude of \mathcal{R} relative to $\delta\varphi$ is reduced, m or λ could be larger than in the normal inflationary setup. However, non-Gaussianity due to second order field perturbations would be correspondingly larger, so the change in the amplitude is restricted to be smaller than one order of magnitude [42]. If the amplitude of \mathcal{R} is instead enhanced, $\delta\varphi$ could be much smaller than usual, and non-Gaussianity from this source would be suppressed from its usual amplitude (which is of order unity). This can in principle be tested observationally, given sufficiently precise observations and control over other sources of non-Gaussianity.

Non-minimal coupling. Let us now consider the case when non-minimal coupling is important. Eliminating $d^2V/d\bar{\varphi}^2$ in the mode equation (2.11) by using (2.10), we have

$$\begin{aligned} & \frac{d^2 u_{\mathbf{k}}}{dN^2} + 3\frac{du_{\mathbf{k}}}{dN} + (3\eta_H + \eta_{H2})u_{\mathbf{k}} \\ & + 6\frac{\xi\bar{\varphi}^2}{1+\xi\bar{\varphi}^2} \left(\xi[4 - 2\epsilon_H + 3\eta_H - \eta_{H2}] - \frac{\bar{\varphi}'}{\mathcal{H}\bar{\varphi}}[2 + 3\xi + (1 + 3\xi)(\epsilon_H - \eta_H)] \right) u_{\mathbf{k}} = 0, \end{aligned} \quad (3.4)$$

where $N \equiv \ln a$. The modes evolve slowly if the absolute value of the dimensionless effective mass term (the factors in front of $u_{\mathbf{k}}$) is much smaller than unity. If the background evolution is such that the slow-roll parameters can be neglected, then smallness of the effective mass requires either that $24\xi^2\bar{\varphi}^2/(1+\xi\bar{\varphi}^2) \ll 1$, or that the factor in parenthesis on the second line of (3.4) is small. Considering the first possibility, if $|\xi|\bar{\varphi}^2 \ll 1$, we also have $\xi^2\bar{\varphi}^2 \ll 1$, and the non-minimal coupling has negligible effect on the dynamics. If instead $|\xi|\bar{\varphi}^2 \gtrsim 1$ and $|\xi| \ll 1$, we have $|\bar{\varphi}| \gg 1$. Vanishing of the factor in parenthesis also leads to the same condition. Thus, if the non-minimal coupling plays a role in inflation, the modes do not evolve slowly, at least for sub-Planckian field values, so the field perturbation spectrum is not close to scale-invariant. According to (2.17)–(2.19) and (2.20), the resulting spectrum of curvature perturbations is not close scale-invariant either. (Matching on a hypersurface that changes the spectrum by the factor $(k/\mathcal{H})^{-4}$ would not compensate to make the spectrum of \mathcal{R} scale-invariant without tuning.) In particular, this rules out inflation with the Standard Model Higgs field [43, 44]. In the super-Planckian case with $|\xi| \ll 1$, the amplitude of primordial perturbations is enhanced relative to the minimally coupled case (if the non-minimal coupling has any role), so the potential has to be even flatter than usual, in addition to other possible problems such as Planck-scale suppressed higher order terms.

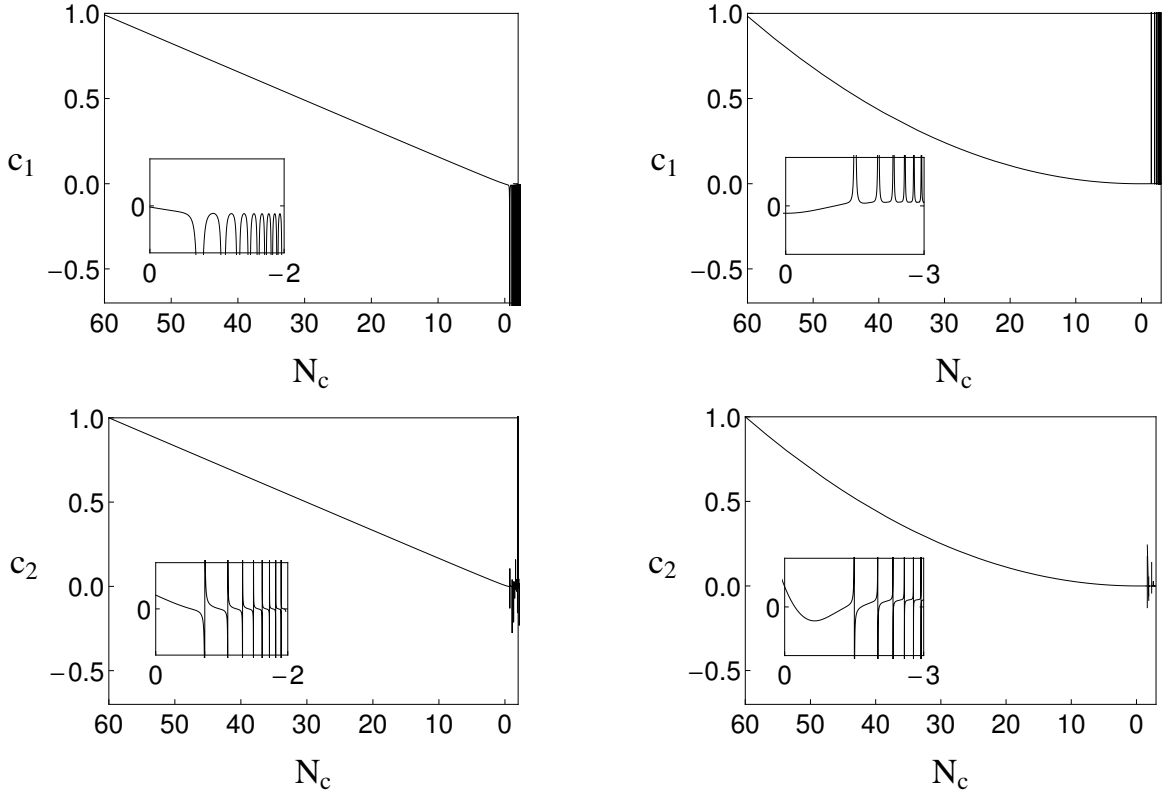


Figure 1. The quantity $c_i \equiv -\mathcal{R}/(\mathcal{H}_{60}\delta\varphi_{60}/\varphi'_{60})$ as a function N_c , the number of e-folds until the end of inflation at collapse; the subscript 60 refers to the terms being evaluated at 60 e-folds until the end of inflation. The top shows c_1 , which corresponds to collapse on the hypersurface of constant $\delta\varphi_{\text{GI}}$. The bottom shows c_2 , which corresponds to collapse on the hypersurface of constant $^{(3)}R$. The left and right sides correspond to the potentials $V = \frac{1}{2}m^2\varphi^2$ and $V = \frac{1}{4}\lambda\varphi^4$, respectively. Insets show evolution after the end of inflation in detail.

Considering the Palatini formalism, in which the metric and the connection are independent variables, would change the equations of motion [45], but this would likely not affect the conclusion. In the slow-roll limit, the effective mass term is only modified by terms that are either quadratic or linear in $\bar{\varphi}'$. Its smallness would again give an independent constraint on the evolution of the background, which is in general not compatible with the background evolution equations. This is in contrast to the minimally coupled inflationary case, in which slow-roll is a sufficient condition for small effective mass.

The reason that inflation driven by non-minimal coupling to gravity does not produce a scale-invariant spectrum is that quantised perturbations of the Ricci scalar R are crucial in the mode equation of motion. In the case when both field and metric perturbations are quantised, slow-roll for the background implies that the effective mass of the perturbations modes is small. In our case, when only the field is quantised, the background Ricci scalar contributes to the background equations, but its perturbations do not contribute to the mode equation, so slow-roll does not guarantee small effective mass. We conclude that in Higgs inflation, a nearly scale-invariant spectrum of scalar perturbations requires quantisation of metric perturbations, unlike in minimally coupled inflation.

4 Conclusions

Inflation in a classical spacetime and the role of collapse. It has been argued that observation of gravitational waves from inflation would provide the first evidence for quantum gravity, because inflationary generation of tensor modes with amplitude close to that of the scalar modes requires quantising metric perturbations. In the usual formalism of inflation, also the scalar modes involve quantised metric perturbations, so gravitational waves do not provide qualitatively new evidence in this regard. However, we do not know which approximation of quantum gravity is valid during inflation, and if it is possible to reproduce the predictions of the usual inflationary formalism without quantising the metric, then gravitational waves would be needed to establish quantisation of gravity.

We have considered inflation in a semiclassical gravity, where the matter is quantised and the metric is classical. We have assumed that the state collapses on a spacelike hypersurface for all wavelengths, in contrast to some previous work where the collapse time depended on the wavelength [23–25]. This means that small wavelength modes collapse before they become squeezed and decohere; it is not clear whether this has observational consequences. Whereas the spectrum for the field perturbation before the collapse is unambiguous, the inherited spectrum of the comoving curvature perturbation depends on the hypersurface of collapse. Selecting the correct hypersurface by some physical principle remains an open question, our main goal was to study whether this setup is tenable in principle. We have considered some simple possibilities and found that for minimally coupled single field inflation models it is possible to recover the usual inflationary results up to leading order in slow-roll parameters, provided the collapse happens during inflation. If the state collapses during preheating, the amplitude of the power spectrum can change by an arbitrary amount. This would change the amplitude of non-Gaussianity due to second order inflaton field perturbations, and there are tight constraints on possible increase of the amplitude, though it can be decreased without limit.

In inflationary models where non-minimal coupling to gravity plays an important role, the situation is different, at least when the field amplitude is sub-Planckian. In that case, metric perturbations play a crucial role in the equation of motion of the mode functions, and it is not possible to generate a nearly scale-invariant spectrum of scalar perturbations without quantising the metric. Thus, models like Higgs inflation require quantum gravity.

In summary, without a detection of gravitational waves or confirmation that non-minimal coupling to gravity plays an important role in inflaton dynamics, successful inflation does not require quantising any part of the metric. Therefore, without the detection of inflationary gravitational waves, we cannot conclude (from tree-level results in inflation) that gravity is quantised.

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